Workshop on

Recent Developments in Mathematical Physics

October 16 - 17, 2019

Osaka Prefecture University

Nakamozu Campus, Building A14 3F Room 315

Schedule

Oct. 16 (Wed)

- **13:30-14:30** Zhanna Kuznetsova (UFABC, São Paulo)
  Low-dimensional $\mathbb{Z}_2 \times \mathbb{Z}_2$ Lie algebras and Lie superalgebras
- **14:40-15:10** Haruka Mori (Kitasato U.)
  Manin Triples for Lie Algebroid Pairs
- **15:20-15:50** Kenta Shiozawa (Kitasato U.)
  Doubled Aspects of Vaisman Algebroid in Para-Hermitian Geometry
- **16:00-16:30** Kosuke Amakawa (Osaka Pref. U.)
  $\mathcal{N}$-Extention of double-graded supersymmetric and superconformal quantum mechanics
- **16:40-17:40** Francesco Toppan (CBPF, Rio de Janeiro)
  A Three-dimensional Superconformal Quantum Mechanics

Oct. 17 (Thu)

- **9:15-9:45** Yuta Nasuda (Tokyo U. of Science, Noda)
  Several approaches toward exact solutions of SUSY QM in the neighborhood of shape invariance

- **9:55-10:25** Satoshi Okumura (Tohoku U.)
  Spectral Properties of a Dirac Operator in the Chiral Quark Soliton Model (Survey)

Special talk on 100th anniversary of Noether's theorem*

- **10:40 - 12:10** Zhanna Kuznetsova (UFABC, São Paulo)
  Emmy Noether: Life and Science. An Invitation to Celebrate 100 Years of Noether's Theorems

*) Emmy Noether's famous paper relating continuous symmetries and conservation laws was published in 1918:

Emmy Noether, "Invariante Variationsprobleme",

Nachrichten der Akademie der Wissenschaften in Göttingen. II.

Mathematisch-Physikalische Klasse (1918) 235-257.

On the occasion of the centenary anniversary (2018) of Noether's profound discovery, Prof. Kuznetsova gave a talk on life and science of Emmy Noether in Brazil a couple of times. Prof. Kuznetsova kindly deliver the talk again in Osaka.

Contact

Naruhiko Aizawa
Department of Physical Science, Graduate School of Science,
Osaka Prefecture University,
Nakamozu Campus, Sakai, Osaka 599-8531
Office: A14 Room 319
Tel: 072 254 9537
E-mail: aizawa[at]p.s.osakafu-u.ac.jp

Abstracts

Kuznetsova: Low-dimensional $\mathbb{Z}_2 \times \mathbb{Z}_2$ Lie algebras and Lie superalgebras

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Lie algebras and superalgebras are the most direct generalizations of $\mathbb{Z}_2$ graded Lie algebras. They are the simplest particular case of color Lie (super) algebras introduced in 1978 by Rittenberg and Wyler.

After the introduction and a brief review of the literature I present a list of all non-equivalent 4-dimensional color Lie algebras and color Lie superalgebras of $\mathbb{Z}_2 \times \mathbb{Z}_2$ type. The properties of the most interesting examples are considered.

Mori: Manin Triples for Lie Algebroid Pairs

The Vaisman algebroid is a kind of algebroid structure. It is defined by an extension of the Courant algebroid. We focus on "the doubled structure" in the Vaisman algebroid. We find that the Vaisman algebroid can be obtained by an analogue of the "Drinfel'd double" of Lie algebroids. This talk is based on arXiv:1901.04777.

Shiozawa: Doubled Aspects of Vaisman Algebroid in Para-Hermitian Geometry

In this talk, we discuss the doubled structure of the Vaisman algebroid in a para-Hermitian geometry. The tangent bundle of a para-Hermitian manifold is decomposed into two eigenbunldes corresponding to the eigenvalues of the para-complex structure. We examine exterior algebras and a para-Dolbeault cohomology on the eigenbundles. We define Lie algebroid structures on the eigenbundles. The Vaisman algebroid is obtained by a double of a pair of Lie algebroids (as explained by Ms. Mori's talk). We provide a geometric realization of the Vaisman algebroid as a double of a pair of Lie algebroids. A para-Hermitian geometry is the geometry of Double Field Theory. We also discuss the physical aspects of this doubled structure. This talk is based on arXiv:1901.04777.

Amakawa: $\mathcal{N}$-Extention of double-graded supersymmetric and superconformal quantum mechanics

In the recent paper, Bruce and Duplij introduced a double-graded version of supersymmetric quantum mechanics (SQM). It is an extension of Lie superalgebraic nature of

$\mathcal{N} = 1$ SQM to a $\mathbb{Z}_2^2$-graded superalgebra. In this talk, two extentions of Bruce-Duplij model are presented : one is $\mathbb{Z}_2^2$-graded SQM of higher values of $\mathcal{N}$, and the other is $\mathbb{Z}_2^2$-graded version of superconformal quantum mechanics. In these extentions, the method which converts a given Lie superalgebra to a $\mathbb{Z}_2^2$-graded superalgebra plays an important role.

Toppan: A Three-dimensional Superconformal Quantum Mechanics

A three-dimensional superconformal quantum mechanics and its associated de Alfaro-Fubini-Furlan deformed oscillator possessing an $sl(2|1)$ dynamical symmetry is constructed. At a coupling parameter b different from zero the Hamiltonian contains a $1/(r^2)$ potential and a spin-orbit (hence, a first-order differential operator) interacting term. At $b = 0$ four copies of undeformed three-dimensional oscillators are recovered. The Hamiltonian gets diagonalized in each sector of total j and orbital l angular momentum (the spin of the system is 1/2). The Hilbert space of the deformed oscillator is given by a direct sum of $sl(2|1)$ lowest weight representations. The selection of the admissible Hilbert spaces at given values of the coupling constant b is discussed. The spectrum of the model is computed. The vacuum energy (as a function of b) consists of a recursive zigzag pattern. The degeneracy of the energy eigenvalues grows linearly up to E equal to b (in proper units) and quadratically for $E > b$. The orthonormal energy eigenstates are expressed in terms of the associated Laguerre polynomials and the spin spherical harmonics. The dimensional reduction of the model to $d = 2$ produces two copies (for b and $-b$, respectively) of the two-dimensional $sl(2|1)$ deformed oscillator. The dimensional reduction to $d = 1$ produces the one-dimensional $D(2, 1; \alpha)$ deformed oscillator, with a determined by b.

Based on the arXiv:1906.11705 paper in collaboration with I.E. Cunha.

Nasuda: Several approaches toward exact solutions of SUSY QM in the neighborhood of shape invariance

It has been extensively investigated for many years how to obtain exact solutions of Schrödinger equations. Supersymmetric quantum mechanics (SUSY QM) leads us to the essential aspects of solvable systems in quantum mechanics [1]. For example, *shape* *invariance* (SI), which is formally described as

$$V_+(x;a_0) = V_-(x;a_1) + R(a_0) \ ,$$

is known as a sufficient condition of exact solvability. SI potentials and several other ones are exactly solvable potentials. However, one rarely finds physically oriented potentials in them. Most potentials in physics are, in fact, non-solvable potentials.

It has shown that there exist "in-between" potentials; we analytically obtain a few of the eigenstates, approximate solutions, or at least some mathematical implications to the numerical methods are suggested. It is interesting to explore analytical methods of obtaining solutions for that kind of systems. Recently, a powerful way of constructing approximate solutions for a potential

$$V(x) = \frac{l(l+1)}{x^2} + ax + bx^2 - \frac{c}{x} \ ,$$

which is not SI, has proposed where the modified concept of SI is applied [2]. We justify this by using the logarithmic perturbation theory [3] and call such potentials "almost shape invariant (ASI)" potentials.

Moreover, we thoroughly examine the following three issues to deal with ASI potentials and discuss the practicabilities and the plausibilities of them.

- Perturbative approach:
  ASI potentials are regarded as the combinations of a SI (super)potential and a perturbation.

- Variational method [4]:
  The superpotential is used to determine a trial function for problems concerning confinment. Then one has to carry out numerical calculation.

- Algebraic property:
  Lie point symmetry is applied to solve different kinds of different equations including the time-dependent Schrödinger equation for the harmonic oscillator. Also, one can find the algebra behind the SI systems [5]. The corresponding approximate algebra [6] might exist for our (approximately solvable) ASI system.

References:

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[4] E. D. Filho and R. M. Ricota, "Supersymmetric variational energies for the confined Coulomb system," Phys. Lett. A **299**, 137 (2002)

[5] C. Rasinariu, J. Mallow and A. Gangopadhyaya, "Exactly solvable problems of quantum mechanics and their spectrum generating algebras: a review," Open Physics **5**, 111 (2007)

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Okumura: Spectral Properties of a Dirac Operator in the Chiral Quark Soliton Model (Survey)

In this talk, I introduce the chiral quark soliton model (CQSM), a low energy effective model in quantum chromodynamics, from the viewpoint of mathematical spectral theory. CQSM was proposed in the 1980s, and whose Hamiltonian is given by a Dirac-type operator with a free Dirac operator perturbed by an isospin term.

The CQSM Hamiltonian differs from the usual Dirac type operator in that the mass term is a matrix-valued function with an effect of an interaction between quarks and the pion field. This fact is also interesting from the viewpoint of operator theory. In particular, the spectrum of CQSM Hamiltonian differs from the free Dirac-type operator in that it has a discrete spectrum between the absolute values of the mass. In addition, by assuming certain appropriate conditions, the CQSM Hamiltonian can be regarded as a generator of supersymmetric quantum mechanics. Furthermore we see the existence of a positive energy ground state or a negative energy ground state for a scaled family of CQSM under appropriate conditions.

The above is what is known so far mainly. However, there is no research that mathematical analysis of this model from the aspect of scattering theory. I would like to investigate the Hamiltonian of CQSM using scattering theory, so I will mention the future prospects.

References:

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[5] M. Reed and B. Simon, "Methods of Modern Mathematical Physics III: Scattering Theory", Academic Press, New York, 1975.

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Kuznetsova: Emmy Noether: Life and Science. An Invitation to Celebrate 100 Years of Noether's Theorems

"The world gave her nothing but obstacles. - She gave us one of the most powerful theorems in physics" (Noether circle at the Perimeter Institute).

The Noether's theorems establish a connection between the two most important ideas in science: conservation laws and symmetries of Nature.

In the first part of the talk we will remember life of Emmy Noether, discussing her private life, oficial career and three different periods in the research. After we will review Noether theorems, symmetries and their impact to different areas of physics up to now. The talk is mostly historical but in this case history and science are inseparable.